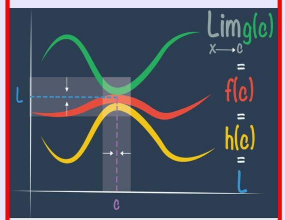
Math 261

Fall 2022

Lecture 35

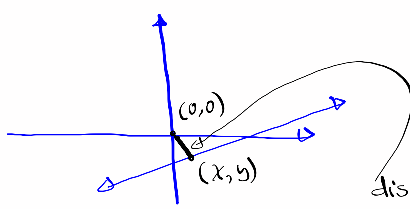


class QZ 9:

Find the point on the line $2x-4y=3$ that is the closest to the Origin.

x	y
0	$-3/4$
$\frac{3}{2}$	0

$$y=\frac{1}{2}x-\frac{3}{4}$$



distance to be minimum.

$$d=\sqrt{(x-0)^2+(y-0)^2}=\sqrt{x^2+\left(\frac{1}{2}x-\frac{3}{4}\right)^2}$$

Minimize $f(x)=x^2+\left(\frac{1}{2}x-\frac{3}{4}\right)^2$

$$f'(x)=2x+2\left(\frac{1}{2}x-\frac{3}{4}\right)\cdot\frac{1}{2} \Rightarrow f'(x)=2x+\frac{1}{2}x-\frac{3}{4}$$

$f'(x)=\frac{5}{2}x-\frac{3}{4}$ $\qquad f'(x)=0 \qquad \frac{5}{2}x-\frac{3}{4}=0$

$f''(x)=\frac{5}{2}>0$ $\qquad$ C.U. $\qquad 10x-3=0 \qquad x=\frac{3}{10}$

$$\left(\frac{3}{10},\ \frac{1}{2}\cdot\frac{3}{10}-\frac{3}{4}\right)=\left(\frac{3}{10},\ -\frac{6}{10}\right)$$

If $\boxed{f(x) \text{ is differentiable at } a}$ then it's cont. at a.

$f'(a)$ exists, $f'(a) = \lim\limits_{x \to a} \dfrac{f(x)-f(a)}{x-a}$

$$\lim_{x \to a} [f(x) - f(a)] = \lim_{x \to a} \left[\frac{f(x)-f(a)}{x-a} \cdot (x-a) \right]$$

$$= \lim_{x \to a} \frac{f(x)-f(a)}{x-a} \cdot \lim_{x \to a} (x-a)$$

$$= f'(a) \cdot (a-a) = 0$$

$$\lim_{x \to a} [f(x) - f(a)] = 0$$

$$\lim_{x \to a} f(x) - \lim_{x \to a} f(a) = 0$$

$$\lim_{x \to a} f(x) - f(a) = 0 \to f(x) \text{ is cont. at } a.$$

$$\lim_{x \to a} f(x) = f(a)$$

3) $f(x) = x^4 - 4x^3$

Polynomial $\to$ Cont. everywhere $\to$ Domain $(-\infty, \infty)$

Y-Int $\to f(0) = 0 \to (0,0)$

x-Int $\to f(x) = 0 \to x^4 - 4x^3 = 0$ $\quad x^3(x-4) = 0$ $\quad x = 0 \quad x = 4$

$(0,0)$, $(4,0)$

3 times

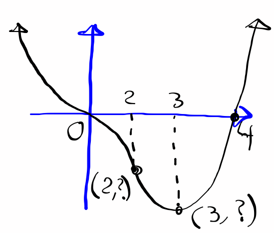
odd # of times

Cross x-axis at $(0,0)$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$ $\quad x = 0$, $x = 3$

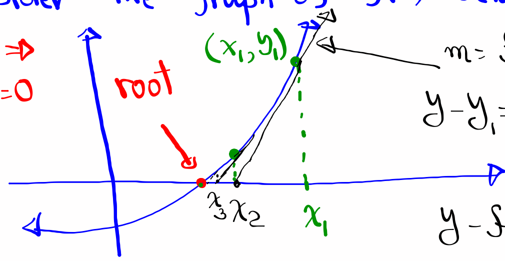
$f''(x) = 12x^2 - 24x = 12x(x-2)$ $\quad x = 0$, $x = 2$

x	$-\infty$	0	2	3	∞
$f'(x)$	$-$	$-$	$-$	$+$	
$f''(x)$	$+$	$-$	$+$	$+$	
$f(x)$	↘	↘	↘	↗	



Consider the graph of $f(x)$ below

root $\Rightarrow$ $f(x)=0$



$m = f'(x_1)$

$y - y_1 = f'(x_1)(x - x_1)$

$y - f(x_1) = f'(x_1)(x - x_1)$

to find x_2 $\rightarrow$ Let $y = 0$

$0 - f(x_1) = f'(x_1)(x_2 - x_1)$

$$x_2 - x_1 = -\frac{f(x_1)}{f'(x_1)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

Newton's Method

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$\vdots$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Solve $\underbrace{x^3 - 2x - 5}_{} = 0$

Polynomial $\rightarrow$ Cont. everywhere

$\rightarrow$ Diff. everywhere

$f(x) = x^3 - 2x - 5$

$f'(x) = 3x^2 - 2$

by Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^3 - 2x_n - 5}{3x_n^2 - 2}$$

let's begin with $x_1 = 2$

$$x_2 = 2 - \frac{2^3 - 2(2) - 5}{3 \cdot 2^2 - 2} = 2.1$$

$$x_3 = 2.1 - \frac{2.1^3 - 2(2.1) - 5}{3 \cdot 2.1^2 - 2} = 2.0946 \approx 2.09$$

$$x_4 = 2.09 - \frac{2.09^3 - 2(2.09) - 5}{3 \cdot 2.09^2 - 2} \approx 2.09456 \approx 2.09$$

Root ≈ 2.09 $\Rightarrow$ check it out

$(2.09)^3 - 2(2.09) - 5 \stackrel{?}{=} 0$

$-.050671$

Estimate $\sqrt[6]{2}$

$x = \sqrt[6]{2}$

$x^6 = 2$

$x^6 - 2 = 0$

$f(x) = x^6 - 2$

$f'(x) = 6x^5$

Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^6 - 2}{6x_n^5}$$

initial guess $x_1 = 1$

$$x_2 = 1 - \frac{1^6 - 2}{6(1)^5} = 1 + \frac{1}{6}$$

$x_2 = 1.1667$

$x_3 = 1.126$

$x_4 = 1.122$

$x_5 = 1.122$

$x_6 = 1.122$

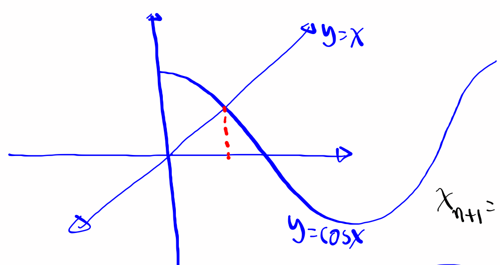
So

$\sqrt[6]{2} \approx 1.122$

from Calc.

Solve $\cos x = x$ in QI.

Let $\cos x - x = 0$



$f(x) = \cos x - x$

$f'(x) = -\sin x - 1$

$$x_{n+1} = x_n - \frac{\cos x_n - x_n}{-\sin x_n - 1}$$

$-(\sin x_n + 1)$

$$x_{n+1} = x_n + \frac{\cos x_n - x_n}{\sin x_n + 1} = \frac{x_n(\sin x_n + 1) + \cos x_n - x_n}{\sin x_n + 1}$$

$$\boxed{x_{n+1} = \frac{x_n \sin x_n + \cos x_n}{\sin x_n + 1}}$$

$x_1 = 1 \to x_2 = .750$

$\to x_3 = .739$

$\to x_4 = .739$

$\to x_5 = .739$

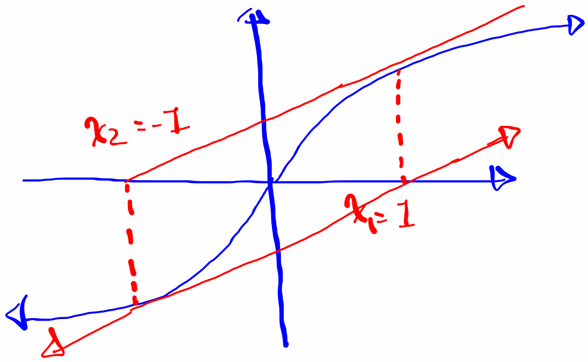
Estimated root $.739$

$\cos .739 - .739 \approx$

$1.4 \times 10^{-4} =$

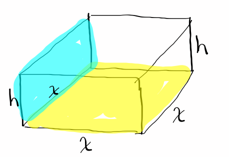
0.00014

Consider the graph of $y = \sqrt[3]{x}$



A box with square base, open top has a Volume of $32{,}000\ cm^3$.

Find dimensions with minimum materials used to make the box.



Volume $= x \cdot x \cdot h$

$32000 = x^2 h \rightarrow h = \frac{32000}{x^2}$

open top

Material needed for

base + Sides

we need to minimize this $\longrightarrow x^2 + 4 \cdot xh$

$$M(x) = x^2 + 4x \cdot \frac{32000}{x^2} = x^2 + \frac{128000}{x}$$

$$M'(x) = 2x - \frac{128000}{x^2}$$

$$2x - \frac{128000}{x^2} = 0$$

$$x^3 = 64000$$

$$x = 40$$

$$M''(x) = 2 + \frac{256000}{x^3} > 0$$

Minimum

$(40, \ )$

$x = 40$

$32000 = x^2 h$

$32000 = 40^2 h \rightarrow \boxed{h = 20}$